Markov Models and Renaissance Music

Re-examining Four-Voice Motets by Josquin

Relationships

- Characters in literature share relationships with one another.
 - How are characters connected?
 - How do the character connections contribute to a narrative?
 - How can those relationships be visualized?
- Pitches, intervals, sounding simultaneities, rhythms share relationships in music:
 - How are pitches related to each other?
 - How are pitches related to intervals?
 - How are pitches related to harmonies?



Relationships

- How are pitches related to rhythms?
- How are any of the musical attributes related to each other?
- How do these relationships contribute to the development of a composition?
- How do these relationships contribute to the idea of styles and genres?



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Networks

- Relationships in prose, video games, and music can be represented in networks.
- Networks can be represented in tables.
- Tables can contain network representations in matrices.
- Tabular data can be graphically represented in network graphs.
- - (Author of the seven volume set The Art of Programming)

The idea of graphically representing network data stems back to Donald Knuth





Network

Les Misérables



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Markov Model

- Introduced by Russian mathematician Andrey Andreyevich Markov
 - A simple chain:
 - Studied sequence of 20,000 letters in A.S. Pushkin's novel verse 'Eugene Onegin'
 - Stationary vowel probability: p = 0.432 (0th order) •
 - p that a vowel is followed by another vowel: p1 = 0.128
 - p that a consonant is followed by a vowel: p2 = 0.633
 - Thus in a Markov chain:
 - p of future state is X_{t+1} (X random variable, t + 1 is time)
 - depends on the current state X_t



State Transitions

- another.
- The task is statistically achieved by creating state transition matrices (STMs).
- another (B).
- At the end of the task a percentage, or p (probability), is assigned to the number of times a transition occurred from A => B, A => A, B => A, B => B.
- The combined transitions can be described as a bigram, or 2-gram, which in turn can be expressed in a STM:

One of the main ideas behind Markov models is how to randomly move from one state to

• A STM keeps a tally of how many times a state is changed from one discrete point (A) to



State Transition Network

A State Transition Network can be visualized in the following way:

	Α	B
A	0.6	0.4
B	0.7	0.3

State Transition Networks with Musical Parameters

- In polyphonic music there are 2-gram STMs of: •
 - 1. melodic successions
 - 2. vertical successions
 - 3. rhythmic successions
- A melodic succession 2-gram can be generated by the movements of:

• STMs can be generated for 3-grams, 4-grams, 5-grams, and any other number of n-grams.

State Transition Networks with Musical Parameters

move to another melodic strand:

• A vertical succession bigram would include:

• Higher order n-grams would include a series of notes (or a melodic strand) to

State Transition Networks with Musical Parameters

• Rhythmic melodic n-grams can be expressed:

- Melodic and vertical n-grams can be combined (VIS-Framework).
- A look at a STM:

 All permutations of melodic, vertical, and rhythmic successions can result in STMs that can be used to identify statistical attributes of a musical style.

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State Transition Matrix of "Josquin's" De profundis Motet

) C#(1)	D(2)	_1 (0)										
	2 (-)	Eb(3)	E(4)	F(5)	F#(6)	G(7)	G#(8)	A(9)	Bb(10)	B(11)	End	Res
47 0	0.2411	0	0.0213	0.0426	0	0.0142	0	0.0603	0	0.2766	0	0.099
0	1	0	0	0	0	0	0	0	0	0	0	0
76 0.0147	0.0637	0	0.2745	0.0441	0	0.0931	0	0.0245	0	0.0490	0.0049	0.063
0	0	0	0	0	0	0	0	0	0	0	0	0
17 0	0.3594	0	0.1094	0.3229	0.0052	0.0156	0	0.0469	0	0	0	0.099
91 0	0.0523	0	0.4302	0.0698	0	0.3779	0	0.0233	0	0	0	0.017
0	0	0	0.1667	0	0.1667	0.5833	0.0833	0	0	0	0	0
97 0	0.0276	0	0.0897	0.2241	0.0276	0.1828	0	0.2690	0	0.0034	0.0069	0.079
0	0	0	0	0	0.3333	0	0.3333	0.3333	0	0	0	0
83 0	0.0601	0	0.0055	0.0656	0	0.4973	0.0055	0.0601	0.0109	0.2350	0	0.021
0	0	0	0	0	0	0	0	1	0	0	0	0
52 0	0.0342	0	0	0	0	0.0822	0	0.3288	0	0.0685	0.0068	0.034
0	0.2500	0	0	0	0	0	0	0	0	0	0	0.750
93 0	0.0625	0	0.0221	0	0	0.1324	0	0.0294	0	0.0147	0	0.639
	 47 0 76 0.0147 0 17 0 17 0 91 0 0 97 0 0 10 1	4700.241101760.01470.0637760.01470.05231700.35949100.05239700.02769700970097009700.027683005200.3429300.0625	4700.24110010760.01470.0637000001700.359409100.052309700.027609700.027608300.060105200.034209300.06250	4700.241100.021310100760.01470.063700.2745760.01470.0637001700.359400.10949100.052300.43029100.027600.16679700.027600.08979700.0601008300.0601005200.0342009300.062500.0221	4700.241100.02130.042601000760.01470.063700.27450.04417000000100.359400.10940.32299100.052300.43020.06989200.027600.48070.22419300.06010009400.06010009500.060100096000009700.06010009800.06010009900.062500091000009300.062500.02210	4700.241100.02130.0426010100000760.01470.063700.27450.044101000000001700.359400.10940.32290.00529100.052300.43020.069809200000.166700.16679400000.02760.08970.22410.0276950000000096000000009700000000980000000099000000009100000000920000000093000000009400000000940000000095000000009500000000950000 <td< 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