Markov Models and Renaissance Music

Re-examining Four-Voice Motets by Josquin

Relationships

- Characters in literature share relationships with one another.
  - How are characters connected?
  - How do the character connections contribute to a narrative?
  - How can those relationships be visualized?
- Pitches, intervals, sounding simultaneities, rhythms share relationships in music:
  - How are pitches related to each other?
  - How are pitches related to intervals?
  - How are pitches related to harmonies?

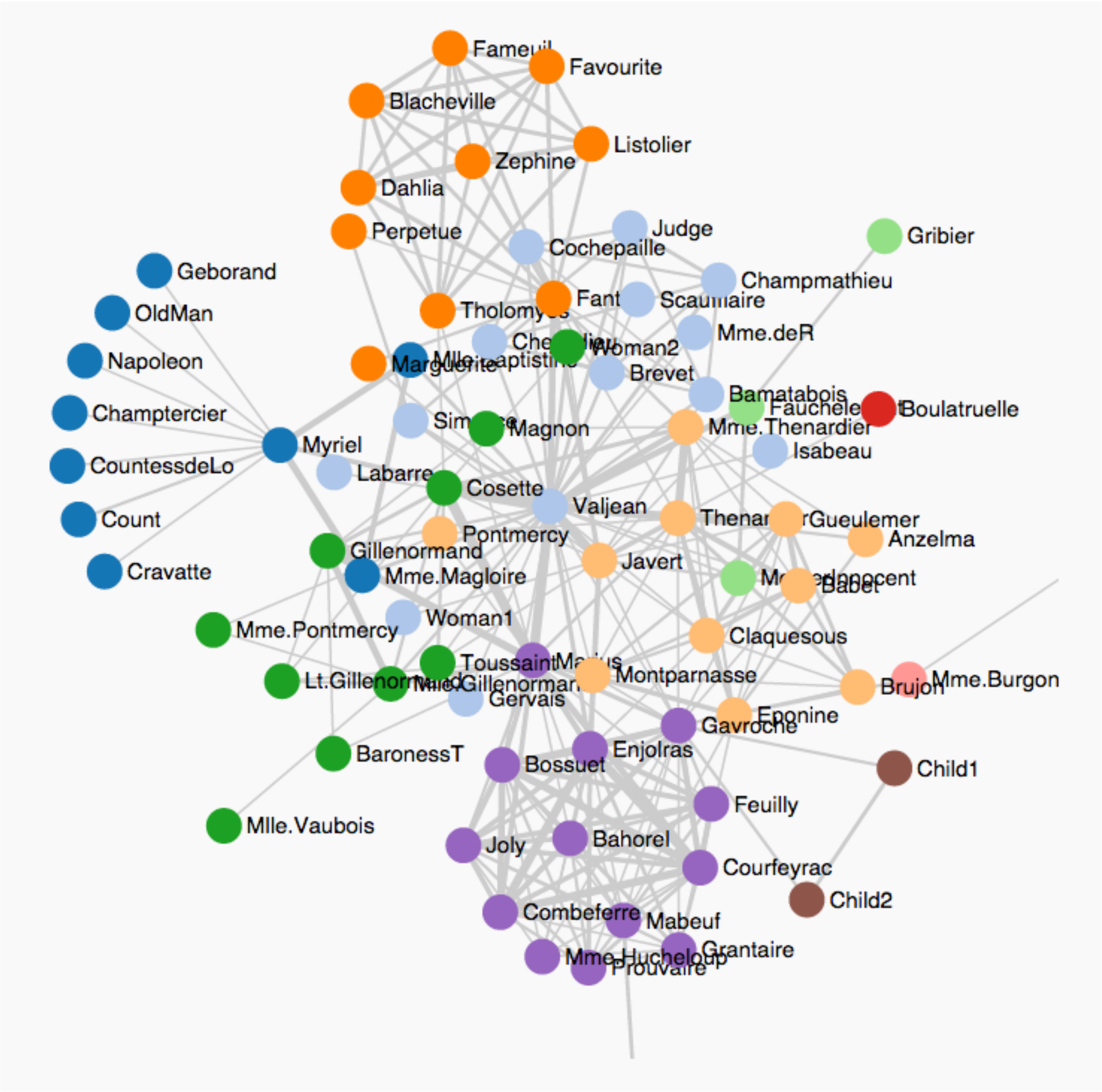
Relationships

- How are pitches related to rhythms?
- How are any of the musical attributes related to each other?
- How do these relationships contribute to the development of a composition?
- How do these relationships contribute to the idea of styles and genres?

Networks

- Relationships in prose, video games, and music can be represented in networks.
- Networks can be represented in tables.
- Tables can contain network representations in matrices.
- Tabular data can be graphically represented in network graphs.
- The idea of graphically representing network data stems back to Donald Knuth
    - (Author of the seven volume set *The Art of Programming*)

Les Misérables



Network

Markov Model

- Introduced by Russian mathematician Andrey Andreyevich Markov
  - A simple chain:
    - Studied sequence of 20,000 letters in A.S. Pushkin's novel verse 'Eugene Onegin'
      - Stationary vowel probability: *p = 0.432* (0th order)
      - *p* that a vowel is followed by another vowel: *p1* = 0.128
      - *p* that a consonant is followed by a vowel: *p2* = 0.633
  - Thus in a Markov chain:
    - *p* of future state is X_{t+1} (X random variable, t + 1 is time)
    - depends on the current state X_t

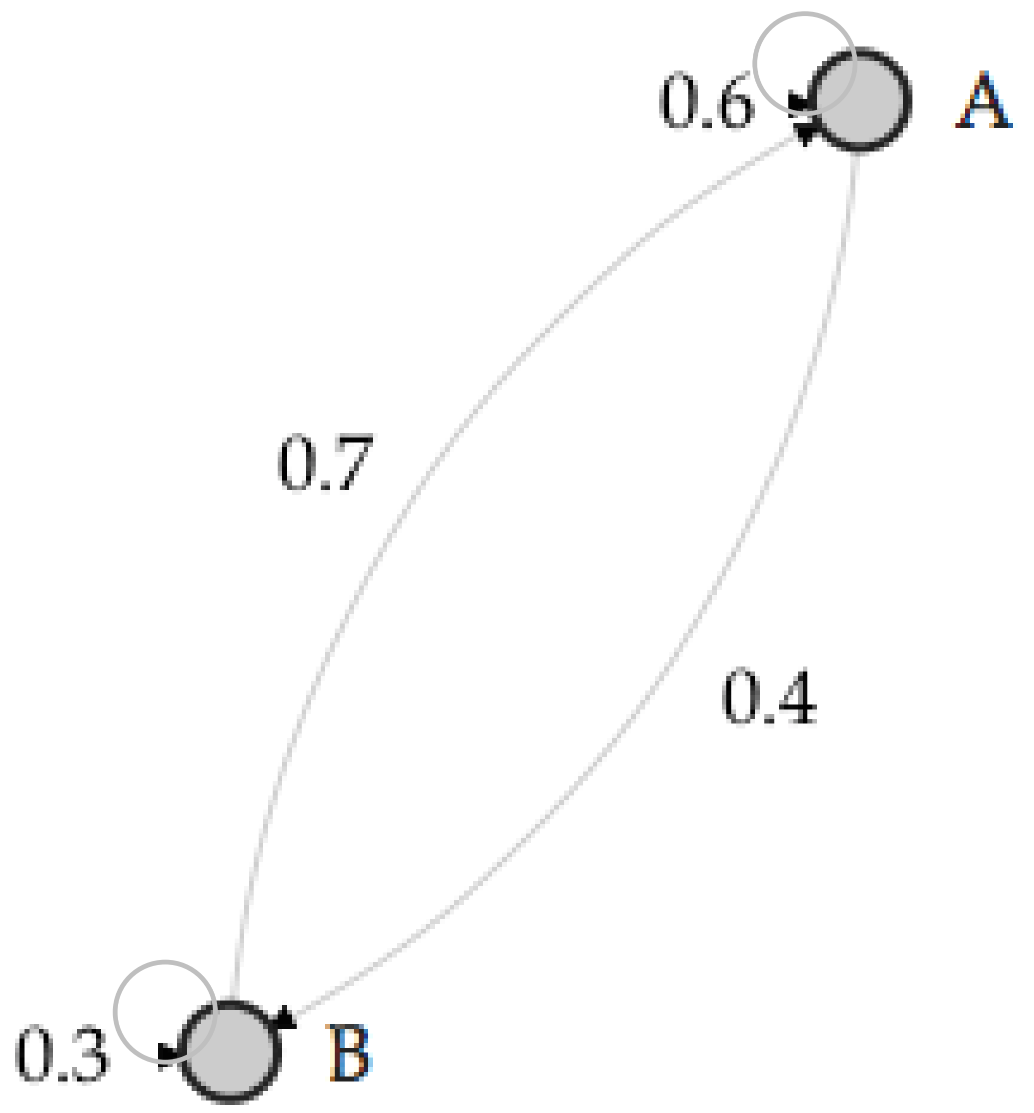
State Transitions

- One of the main ideas behind Markov models is how to randomly move from one state to another.
- The task is statistically achieved by creating state transition matrices (STMs).
- A STM keeps a tally of how many times a state is changed from one discrete point (A) to another (B).
- At the end of the task a percentage, or *p* (*probability*), is assigned to the number of times a transition occurred from A => B, A => A, B => A, B=> B.
- The combined transitions can be described as a bigram, or 2-gram, which in turn can be expressed in a STM:

State Transition Network

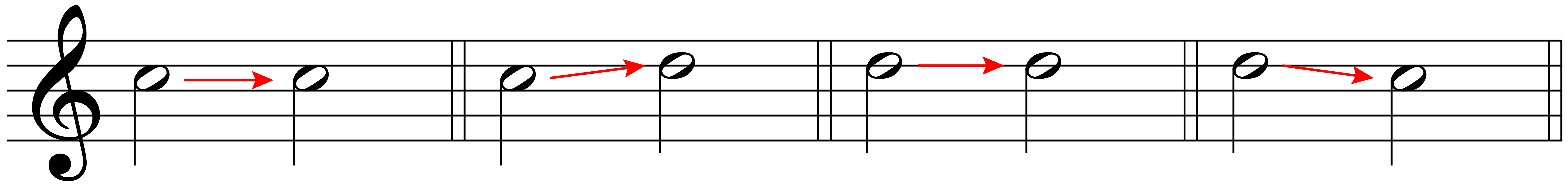
- A State Transition Network can be visualized in the following way:

	A	B
A	0.6	0.4
B	0.7	0.3



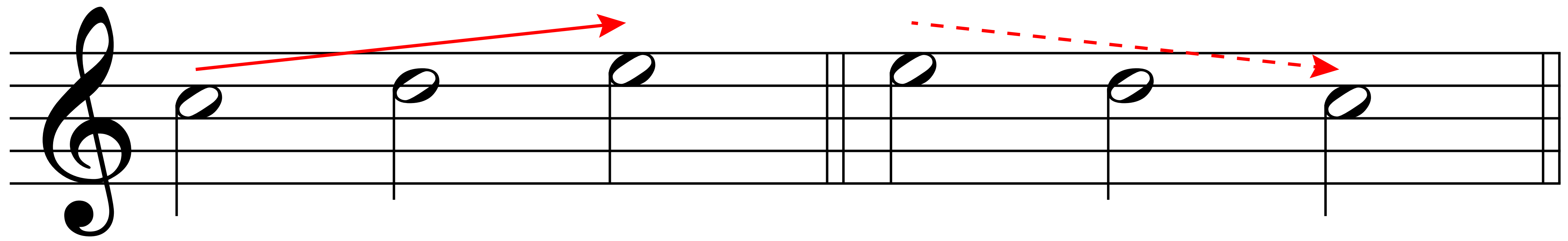
State Transition Networks with Musical Parameters

- In polyphonic music there are 2-gram STMs of:
    1. melodic successions
    2. vertical successions
    3. rhythmic successions
- STMs can be generated for 3-grams, 4-grams, 5-grams, and any other number of n-grams.
- A melodic succession 2-gram can be generated by the movements of:

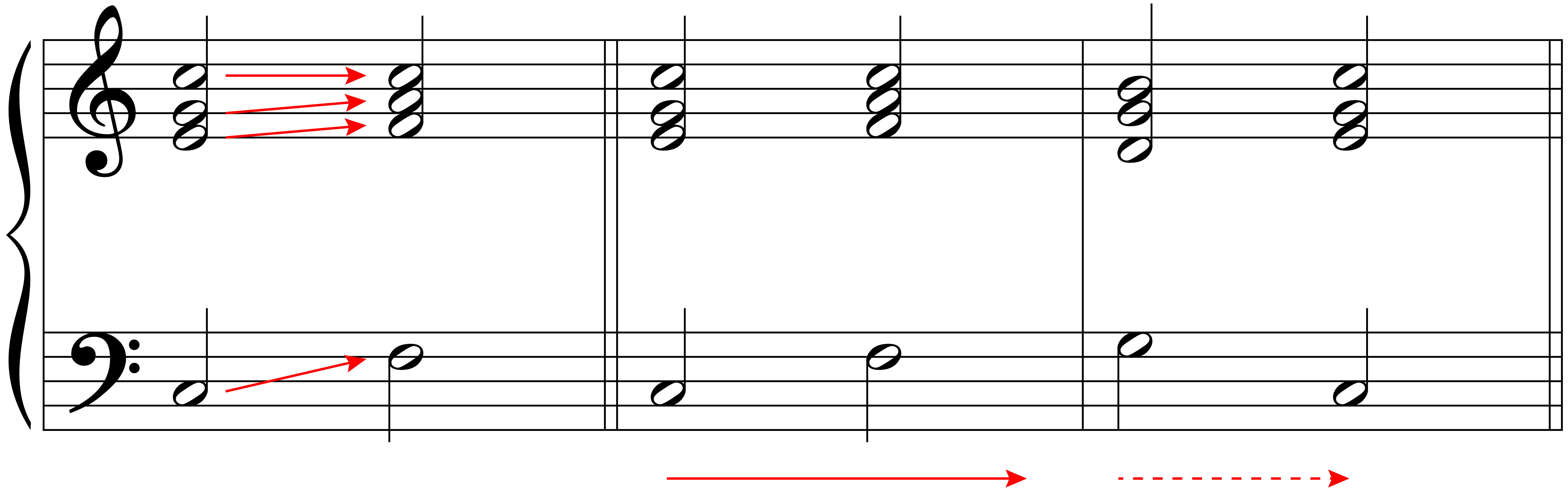


State Transition Networks with Musical Parameters

- Higher order n-grams would include a series of notes (or a melodic strand) to move to another melodic strand:

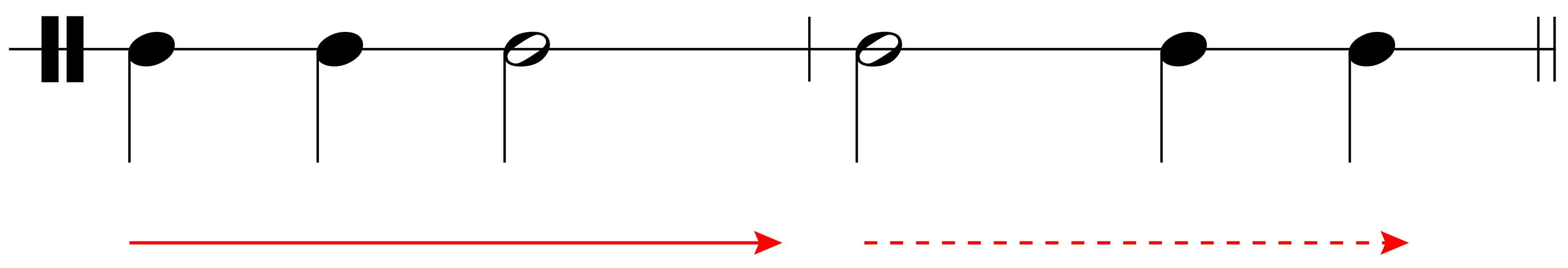


- A vertical succession bigram would include:



State Transition Networks with Musical Parameters

- Rhythmic melodic n-grams can be expressed:



- Melodic and vertical n-grams can be combined (VIS-Framework).

- All permutations of melodic, vertical, and rhythmic successions can result in STMs that can be used to identify statistical attributes of a musical style.

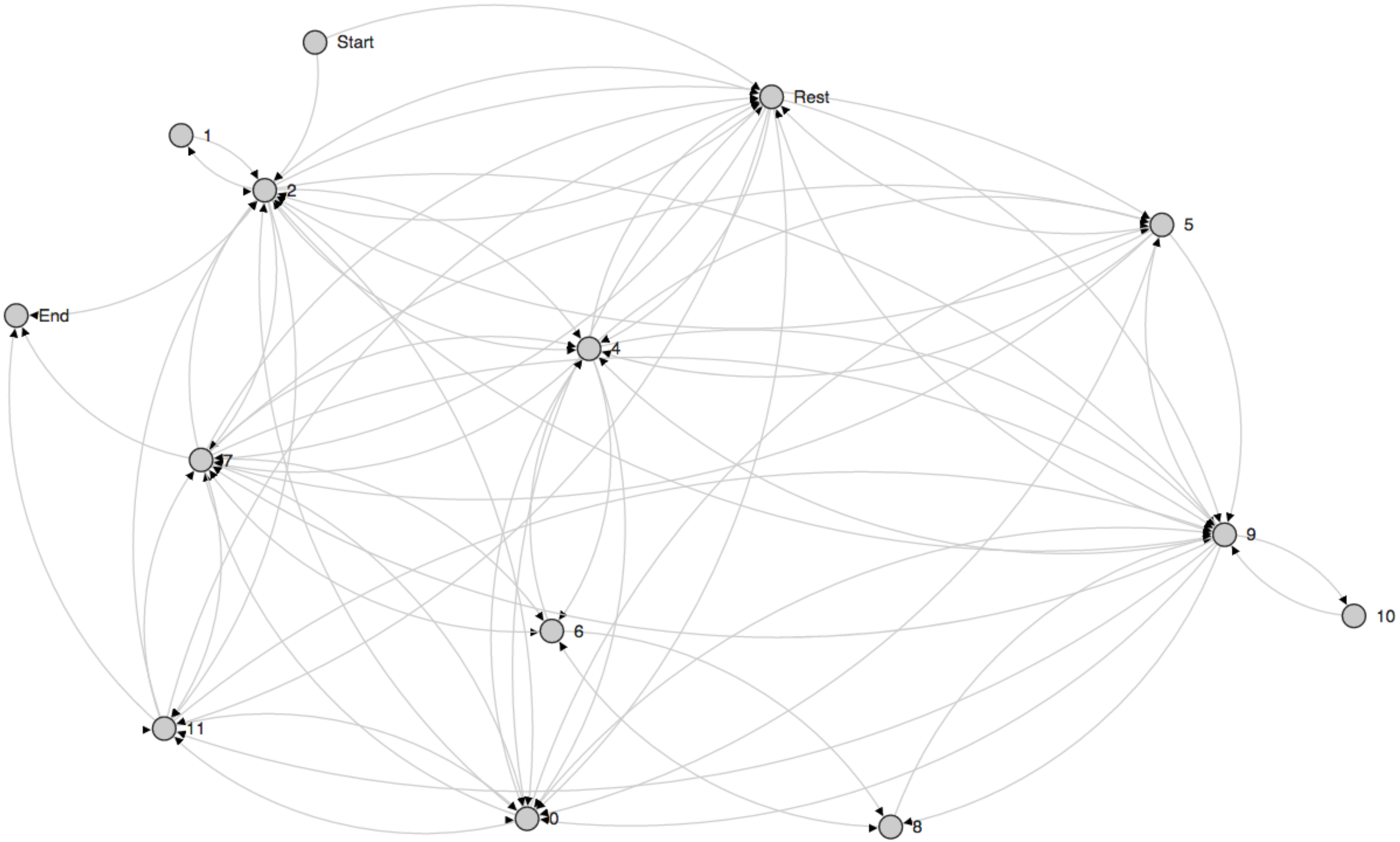
- A look at a STM:

State Transition Matrix of “Josquin’s" *De profundis Motet*

From → To

PCs	C(0)	C#(1)	D(2)	Eb(3)	E(4)	F(5)	F#(6)	G(7)	G#(8)	A(9)	Bb(10)	B(11)	End	Rest
C(0)	**0.2447**	0	**0.2411**	0	0.0213	0.0426	0	0.0142	0	0.0603	0	**0.2766**	0	0.0993
C#(1)	0	0	1	0	0	0	0	0	0	0	0	0	0	0
D(2)	**0.3676**	0.0147	0.0637	0	**0.2745**	0.0441	0	0.0931	0	0.0245	0	0.0490	0.0049	0.0637
Eb(3)	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E(4)	0.0417	0	**0.3594**	0	**0.1094**	**0.3229**	0.0052	0.0156	0	0.0469	0	0	0	0.0990
F(5)	0.0291	0	0.0523	0	**0.4302**	0.0698	0	**0.3779**	0	0.0233	0	0	0	0.0174
F#(6)	0	0	0	0	**0.1667**	0	**0.1667**	**0.5833**	0.0833	0	0	0	0	0
G(7)	0.0897	0	0.0276	0	0.0897	**0.2241**	0.0276	**0.1828**	0	**0.2690**	0	0.0034	0.0069	0.0793
G#(8)	0	0	0	0	0	0	**0.3333**	0	**0.3333**	**0.3333**	0	0	0	0
A(9)	0.0383	0	0.0601	0	0.0055	0.0656	0	**0.4973**	0.0055	0.0601	0.0109	**0.2350**	0	0.0219
Bb(10)	0	0	0	0	0	0	0	0	0	1	0	0	0	0
B(11)	**0.4452**	0	0.0342	0	0	0	0	0.0822	0	**0.3288**	0	0.0685	0.0068	0.0342
Start	0	0	**0.2500**	0	0	0	0	0	0	0	0	0	0	**0.7500**
Rest	0.0993	0	0.0625	0	0.0221	0	0	**0.1324**	0	0.0294	0	0.0147	0	**0.6397**

From → To


Start
Rest
1
2
5
End
4
7
9
10
6
11
0
8

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