Markov Models and Renaissance Music

Re-examining Four-Voice Motets by Josquin
Relationships

• Characters in literature share relationships with one another.
  • How are characters connected?
  • How do the character connections contribute to a narrative?
  • How can those relationships be visualized?

• Pitches, intervals, sounding simultaneities, rhythms share relationships in music:
  • How are pitches related to each other?
  • How are pitches related to intervals?
  • How are pitches related to harmonies?
Relationships

• How are pitches related to rhythms?

• How are any of the musical attributes related to each other?

• How do these relationships contribute to the development of a composition?

• How do these relationships contribute to the idea of styles and genres?
Networks

• Relationships in prose, video games, and music can be represented in networks.

• Networks can be represented in tables.

• Tables can contain network representations in matrices.

• Tabular data can be graphically represented in network graphs.

• The idea of graphically representing network data stems back to Donald Knuth
  • (Author of the seven volume set *The Art of Programming*)
Markov Model

• Introduced by Russian mathematician Andrey Andreyevich Markov

  • A simple chain:
    • Studied sequence of 20,000 letters in A.S. Pushkin’s novel verse ‘Eugene Onegin’
      • Stationary vowel probability: $p = 0.432$ (0th order)
      • $p$ that a vowel is followed by another vowel: $p_1 = 0.128$
      • $p$ that a consonant is followed by a vowel: $p_2 = 0.633$

  • Thus in a Markov chain:
    • $p$ of future state is $X_{t+1}$ (X random variable, $t + 1$ is time)
    • depends on the current state $X_t$
State Transitions

• One of the main ideas behind Markov models is how to randomly move from one state to another.

• The task is statistically achieved by creating state transition matrices (STMs).

• A STM keeps a tally of how many times a state is changed from one discrete point (A) to another (B).

• At the end of the task a percentage, or $p$ (probability), is assigned to the number of times a transition occurred from $A \Rightarrow B$, $A \Rightarrow A$, $B \Rightarrow A$, $B \Rightarrow B$.

• The combined transitions can be described as a bigram, or 2-gram, which in turn can be expressed in a STM:
State Transition Network

- A State Transition Network can be visualized in the following way:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>B</td>
<td>0.7</td>
<td>0.3</td>
</tr>
</tbody>
</table>
State Transition Networks with Musical Parameters

- In polyphonic music there are 2-gram STMs of:
  1. melodic successions
  2. vertical successions
  3. rhythmic successions

- STMs can be generated for 3-grams, 4-grams, 5-grams, and any other number of n-grams.

- A melodic succession 2-gram can be generated by the movements of:
State Transition Networks with Musical Parameters

- Higher order n-grams would include a series of notes (or a melodic strand) to move to another melodic strand:

- A vertical succession bigram would include:
State Transition Networks with Musical Parameters

- Rhythmic melodic n-grams can be expressed:

- Melodic and vertical n-grams can be combined (VIS-Framework).

- All permutations of melodic, vertical, and rhythmic successions can result in STMs that can be used to identify statistical attributes of a musical style.

- A look at a STM:
State Transition Matrix of “Josquin’s” *De profundis* Motet

<table>
<thead>
<tr>
<th>From</th>
<th>C(0)</th>
<th>C#(1)</th>
<th>D(2)</th>
<th>Eb(3)</th>
<th>E(4)</th>
<th>F(5)</th>
<th>F#(6)</th>
<th>G(7)</th>
<th>G#(8)</th>
<th>A(9)</th>
<th>Bb(10)</th>
<th>B(11)</th>
<th>End</th>
<th>Rest</th>
</tr>
</thead>
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<td></td>
<td>0.2411</td>
<td></td>
<td>0.0213</td>
<td>0.0426</td>
<td>0.0142</td>
<td>0.0603</td>
<td>0.2766</td>
<td>0.0993</td>
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<td>0.2447</td>
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<td>0.0142</td>
<td>0.0603</td>
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<td>0.0993</td>
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<tr>
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<td>Eb(3)</td>
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