Key-Finding Based on a Hidden Markov Model and Key Profiles

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An algorithm for computing local and global musical keys from symbolic and audio sources

Similar performance for symbolic and audio sources (according to our evaluation)

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- Global musical keys
 - ♦ State-of-the-art performance in **symbolic** inputs
 - ♦ State-of-the-art Decent performance in audio inputs

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 - ♦ State-of-the-art Decent performance in audio inputs
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 - ♦ Segments the piece in its underlying modulations tonicizations secondary dominants local keys

How does it work?

Using Hidden Markov Models (HMMs)

♦ Very good for modelling processes with *steady* behaviors

An Introduction to Hidden Markov Models, 1986

[...] Many real world processes seem to manifest a rather sequentially changing behavior; the properties of the process are usually held pretty **steadily**, except for minor fluctuations, for a certain period of time (or a number of the abovementioned duration units), and then, at certain instances, **change** (gradually or rapidly) to another set of properties. The opportunity for more efficient modeling [such processes] can be exploited if we can first **identify** these periods of rather steadily behavior, and then are willing to **assume** that the temporal variations within each of these steady periods are, in a sense, **statistical**. A more efficient representation may then be obtained by using a common short time model for **each** of the steady, or well-behaved parts of the signal, along with some characterization of **how** one such period evolves to the next [...]

(Rabiner and Juang, 1986)

How does it work?

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Parameters of an HMM

- ♦ Hidden States (steady behaviors)
- Observations
- ♦ Emission Probability Distributions
- ♦ Transition Probability Distributions
- ♦ Initial State Probability Distribution

How does it work?

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Key-Finding HMM

♦ Hidden States (steady behaviors)
Major and minor keys

♦ Observations Notes (pitch classes)

♦ Emission Probability Distributions: Key profiles

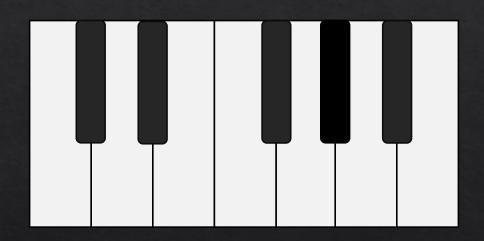
♦ Transition Probability Distributions Weber's tonal areas

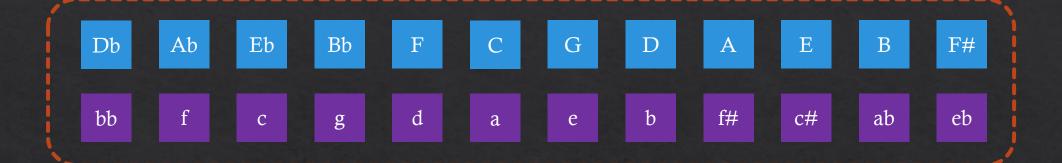
Initial State Probability Distribution Uniform (any key is just as likely at the beginning)

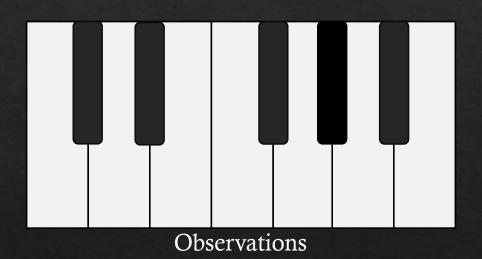
- 1. Observations and hidden states are defined
- 2. The initial probability distribution determines the first hidden state
- 3. After that, for every time step in the process
 - 1. The hidden state *emits* an observation
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 - 3. Repeat for the next time step

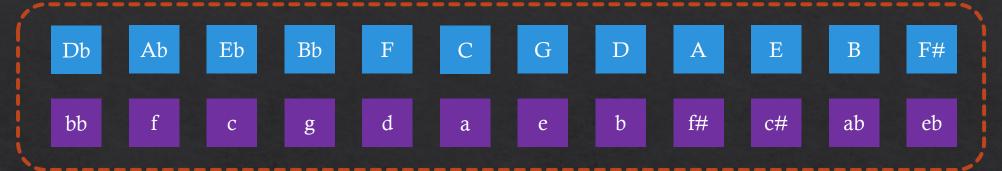
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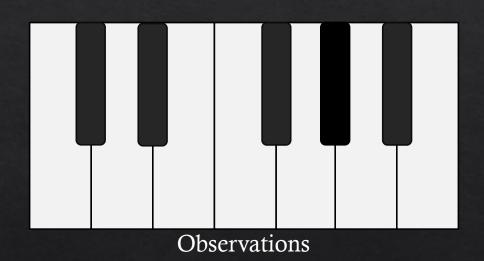
Ab Eb Bb F# Db b f# c# d ab bb g a eb C e





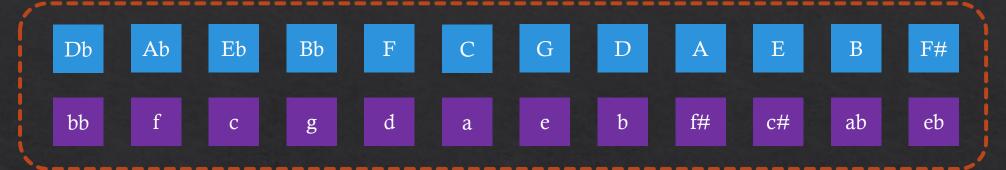


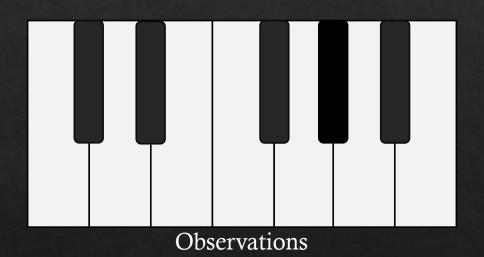


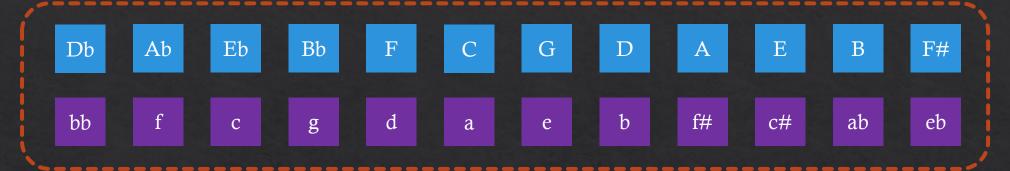


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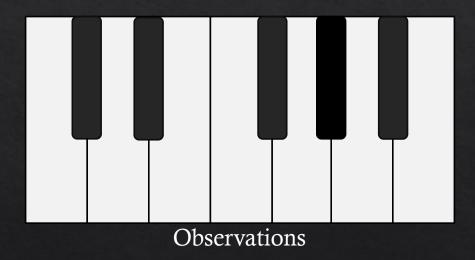
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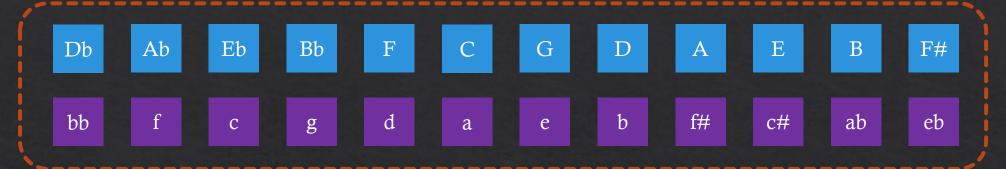


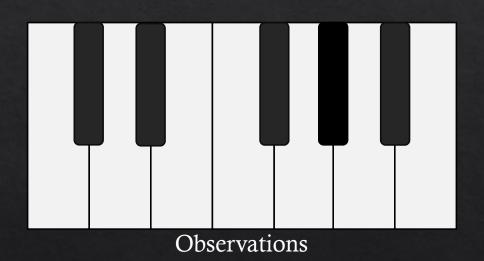




Initial probability distribution

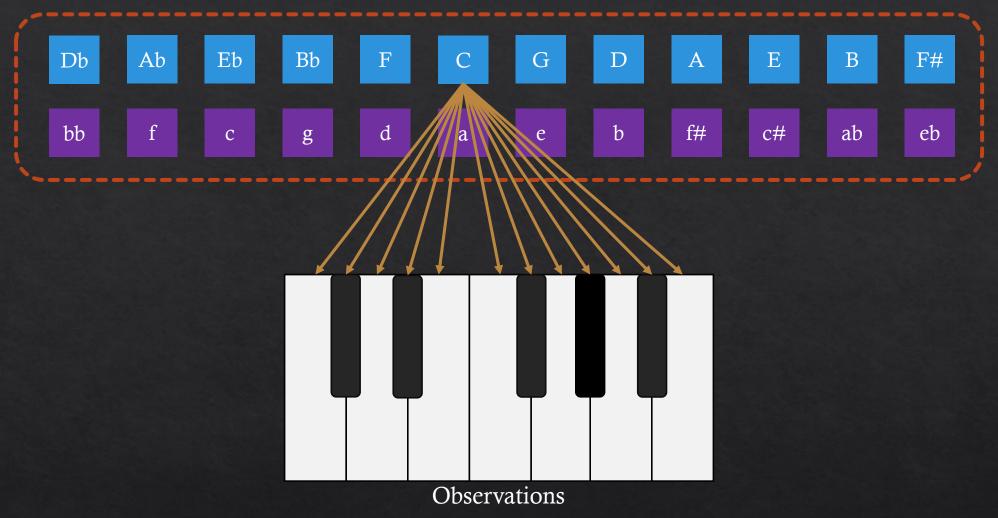


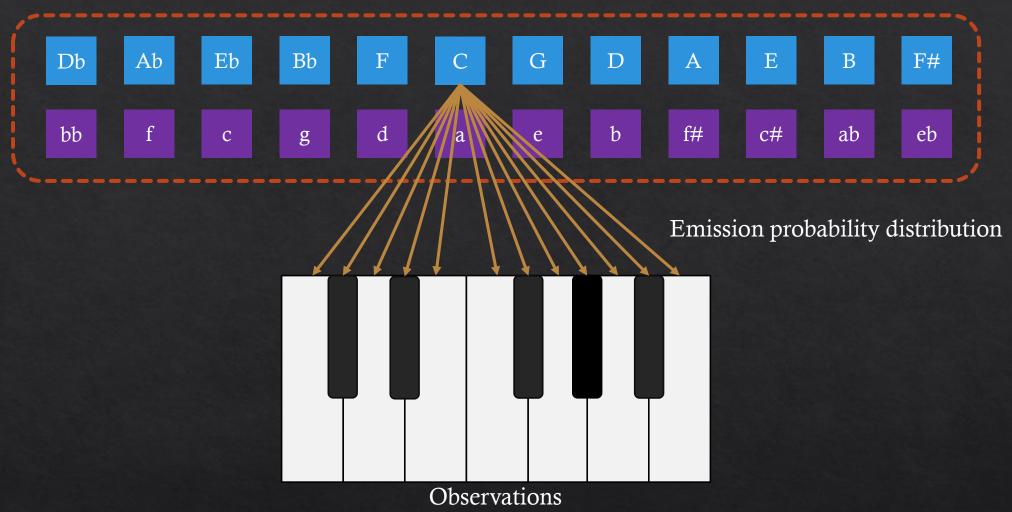




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Key profiles

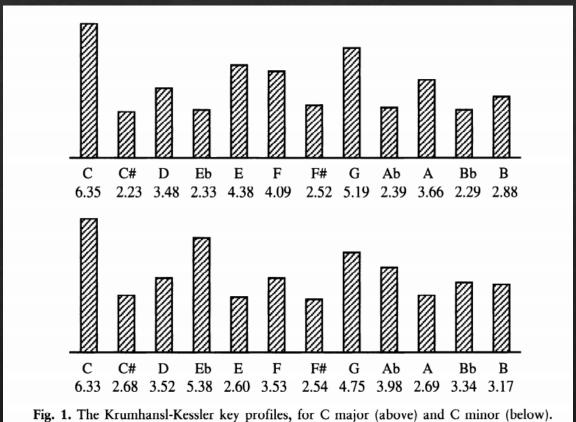


Fig. 1. The Krumhansl-Kessler key profiles, for C major (above) and C minor (below). (Data from Krumhansl, 1990, p. 80.)

Key profiles

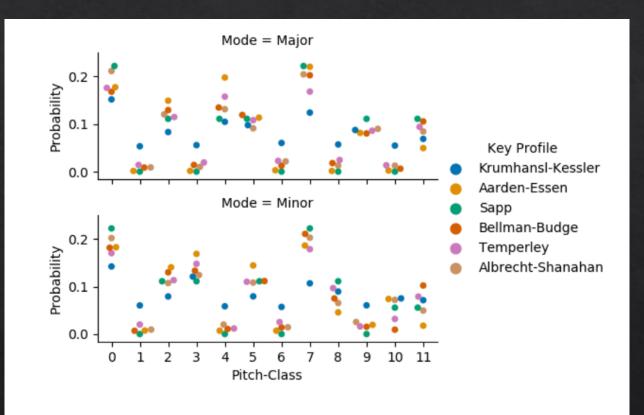
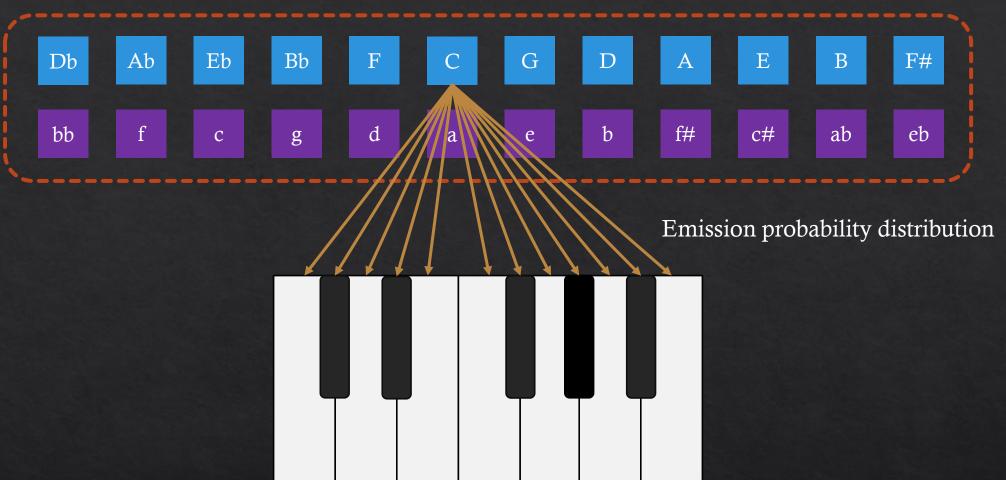
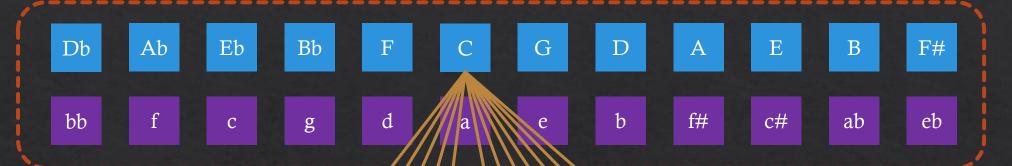


Figure 3: Probabilities of *emitting* a pitch class in C Major (top) and C Minor (bottom), obtained from six key profiles



Observations



Emission probability distribution

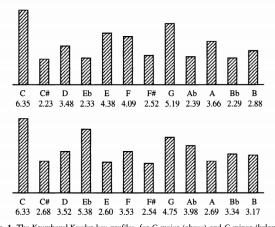
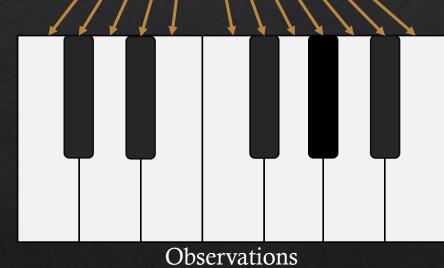
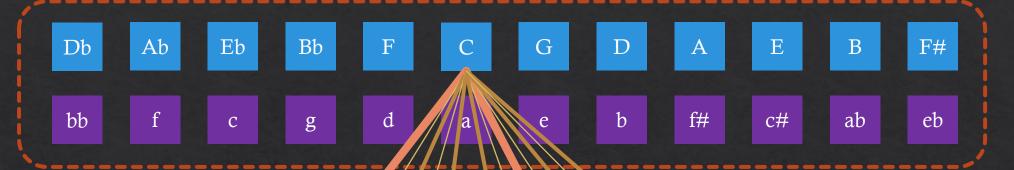


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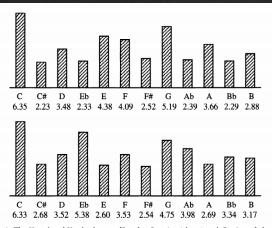
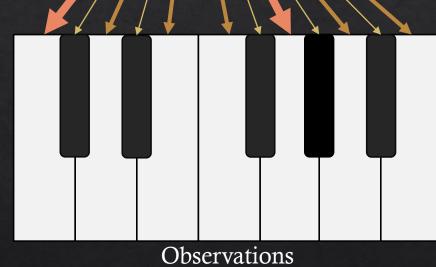
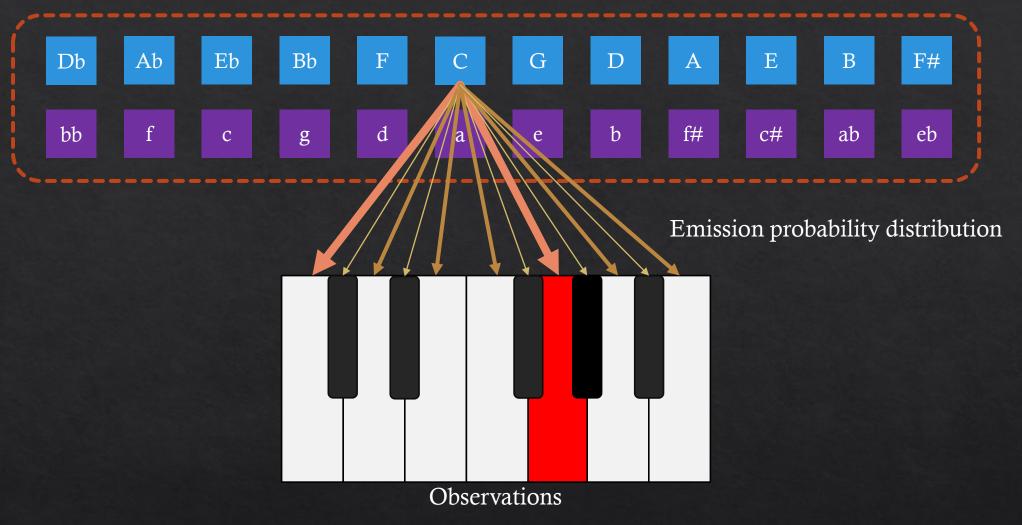


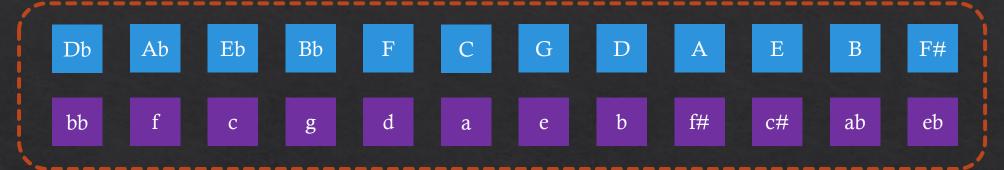
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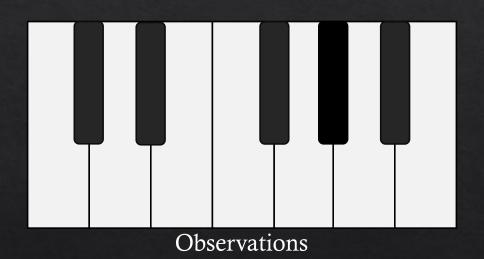


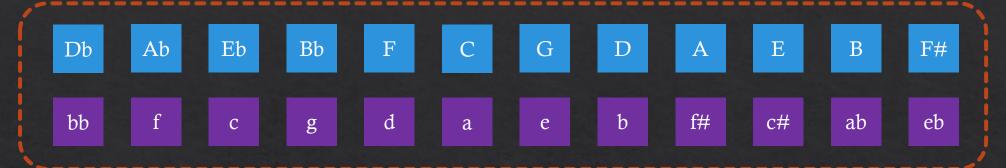


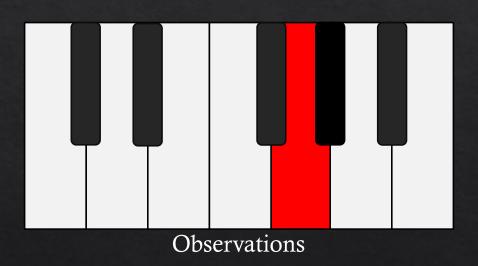
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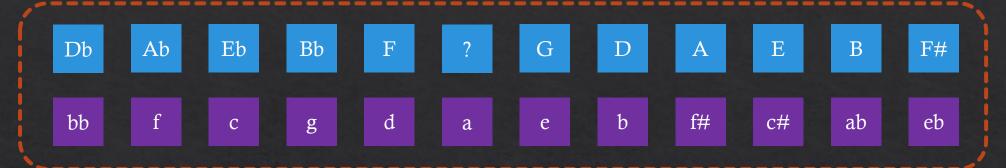


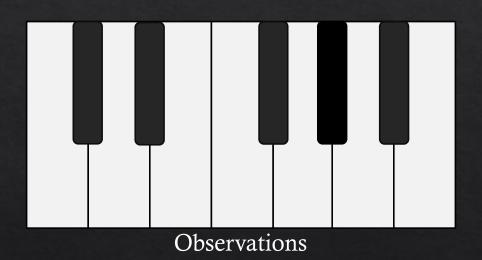




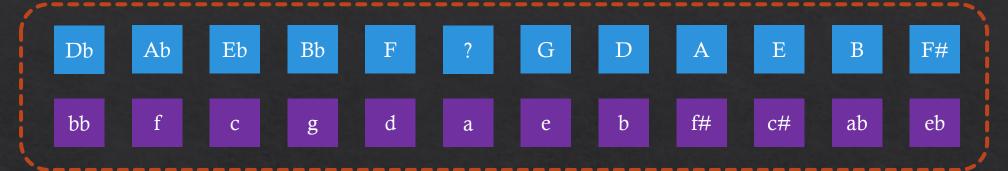


Hidden States





Hidden States



Transition probability distribution



Key transition matrix

A well-known table of related keys (Temperley, Krumhansl, Schoenberg, Weber)

Table 1: Matrix of neighbouring keys. Column-wise, the keys follow the *circle-of-fifths*. Row-wise, each key is surrounded by its relative and parallel major (or minor) keys

Α#	a#	C#	c#	Е	e	G	g	Вδ
$\mathrm{D}\sharp$	$\mathrm{d}\sharp$	F♯	f#	A	a	C	c	Εb
G#	$g \sharp$	В	b	D	d	F	f	Ab
C#	c#	E	e	G	g	Bb	bb	$\mathrm{D}\flat$
F#	f#	Α	a	C	c	Eb	eb	Gb
В	b	D	d	F	f	Αb	ab	Cb
E	e	G	g	Bb	bb	$\mathrm{D}\flat$	db	Fb
A	a	C	c	Εb	eb	Gb	$\mathbf{g}\mathbf{b}$	Bbb
D	d	F	f	Αb	$\mathbf{a} \mathbf{b}$	Cb	cb	Ebb

```
C c Eb eb Gb gb
       Bb bb Db db
  G
     g
b
  D
     d F f Ab ab
  A
     a C c Eb eb
         g Bb bb
     e G
  E
  В
    b
       D
          d F
     f# A a C
d# F#
```

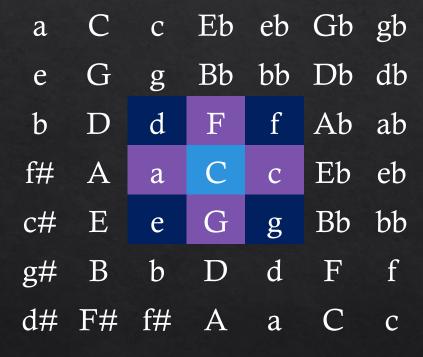
a	C	С	Eb	eb	Gb	gb
e	G	g	Bb	bb	Db	db
b	D	d	F	f	Ab	ab
f#	A	a	C	С	Eb	eb
c#	E	e	G	g	Bb	bb
g#	В	b	D	d	F	f
d#	F#	f#	A	a	C	C

```
C c Eb eb Gb gb
        Bb bb Db db
   G
      g
b
  D
     d
        F
           f Ab ab
           c Eb eb
f#
  A
        C
      a
        G
             Bb bb
  E
           g
      e
  В
     b
       D
           d F
     f#
d# F#
        A a C
```

Group	1	2	3	4	5	6	7	8	9
Keys	С	F	d	D	Е	Db	eb	c#	F#
		G	е	Eb	Ab	В	f#	ab	
		а	f	Α	bb				
		С	g	Bb	b				

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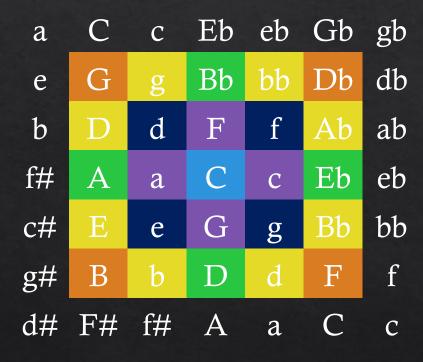
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		а	f	Α	bb				
		С	g	Bb	b				

a	C	C	Eb	eb	Gb	gb
e	G	g	Bb	bb	Db	db
b	D	d	F	f	Ab	ab
f#	A	a	C	С	Eb	eb
c#	E	e	G	g	Bb	bb
g#	В	b	D	d	F	f
d#	F#	f#	A	a	C	C

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		С	g	Bb	b				

a	C	С	Eb	eb	Gb	gb
e	G	g	Bb	bb	Db	db
b	D	d	F	f	Ab	ab
f#	A	a	C	С	Eb	eb
c#	E	e	G	g	Bb	bb
g#	В	Ъ	D	d	F	f
d#	F#	f#	A	a	C	C

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b	D	d	F	f	Ab	ab
f#	A	a	C	С	Eb	eb
c#	E	e	G	g	Bb	bb
g#	В	Ъ	D	d	F	f
d#	F#	f#	A	a	C	С

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a	C	С	Eb	eb	Gb	gb
e	G	g	Bb	bb	Db	db
Ъ	D	d	F	f	Ab	ab
f#	A	a	C	С	Eb	eb
c#	E	e	G	g	Bb	bb
g#	В	Ъ	D	d	F	f
d#	F#	f#	A	a	C	С

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a	C	С	Eb	eb	Gb	gb
e	G	g	ВЪ	bb	Db	db
Ъ	D	d	F	f	Ab	ab
f#	A	a	C	С	Eb	eb
c#	Е	e	G	g	ВЪ	bb
g#	В	Ъ	D	d	F	f
d#	F#	f#	A	a	C	C

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a	C	С	Eb	eb	Gb	gb
e	G	g	Вь	bb	Db	db
b	D	d	F	f	Ab	ab
f#	A	a	C	С	Eb	eb
c#	Е	e	G	g	Bb	bb
g#	В	Ъ	D	d	F	f
d#	F#	f#	A	a	C	С

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		а	f	Α	bb				
		С	g	Bb	b				

$$1 = p + 2p + 3p + 4p + 5p + 6p + 7p + 8p + 9p$$

$$1 = p(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9)$$

$$1 = 45p$$

$$p = 1/45$$

	rai	tio	ratio	ratio	ratio	ratio
Group	1	2	3	4	5	
Keys	С	F	d	D	Е	
		G	е	Eb	Ab	
		а	f	А	bb	
		С	g	Bb	b	

	ra	tio i	ratio	ratio	ratio	ratio
Group	1	2	3	4	5	
Keys	С	F	d	D	Е	
		G	е	Eb	Ab	
		а	f	А	bb	
		С	g	Bb	b	

for
$$ratio = 2$$

$$1 = 2^{0}p + 2^{1}p + 2^{2}p + 2^{3}p + 2^{4}p + 2^{5}p + 2^{6}p + 2^{7}p + 2^{8}p$$

$$1 = p(1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256)$$

$$p = 1/511 = 0.002$$

Transition probability distribution

According to three different ratios

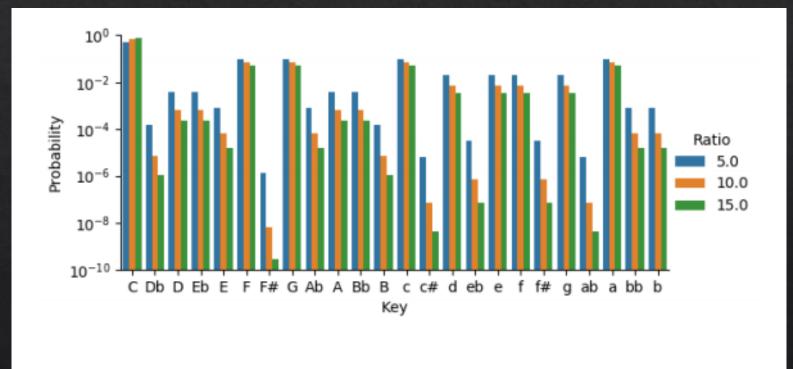
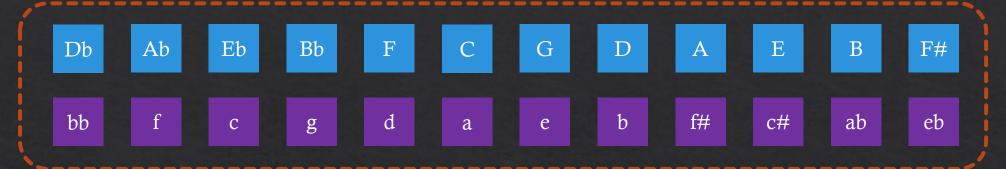
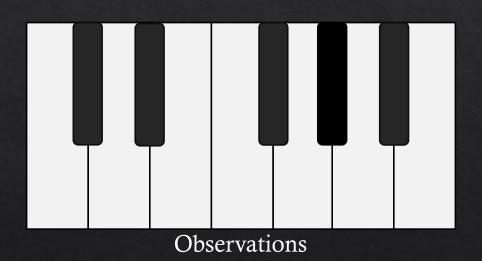
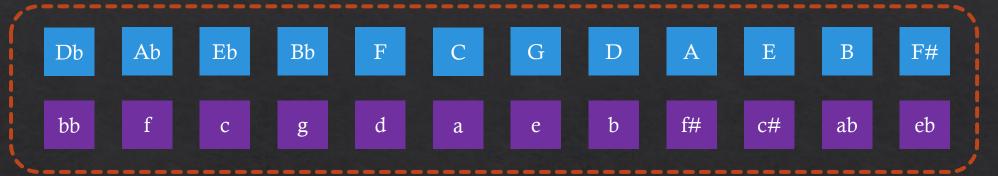


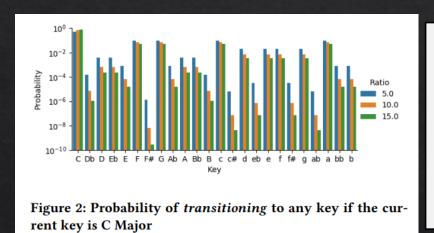
Figure 2: Probability of transitioning to any key if the current key is C Major

Hidden States

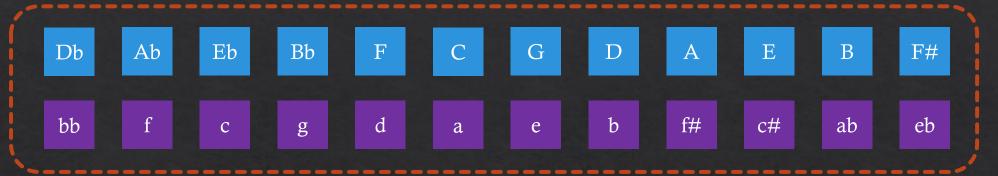


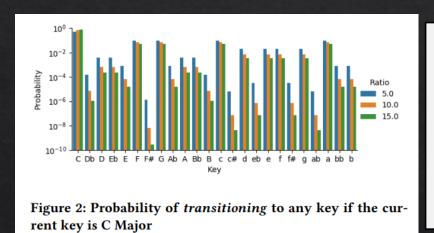














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Analyzing instead of generating

- ◆ In a musical piece, we are given the observations (notes that play over time)
- Using the same parameters described before, we can reverse the process and compute the sequence of *hidden states* (keys) that have the largest probability of explaining the sequence of observations
- ♦ We call that sequence of hidden states, the *local keys* of the piece

Demo

justkeydding

https://github.com/napulen/justkeydding

Handling symbolic inputs

- Parsing notes from bottom to top (when possible)
- Only pitch class is utilized (no spelling nor octave)
- Vertical relationship between notes and duration of the notes are also discarded

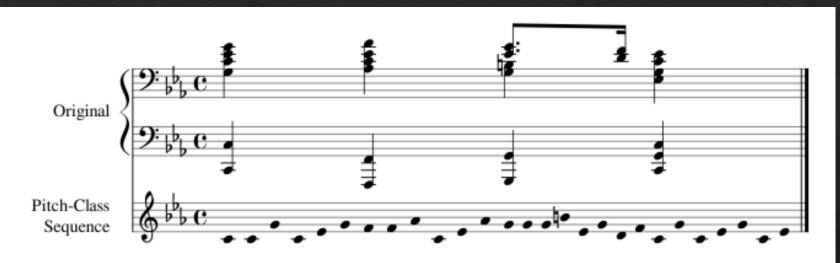
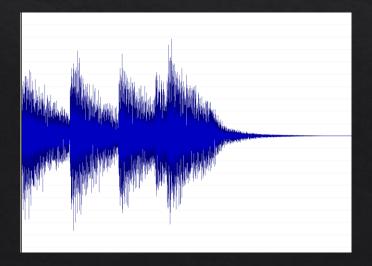


Figure 1: Pitch-class sequence from the first measure of Chopin's Op. 28 No. 20

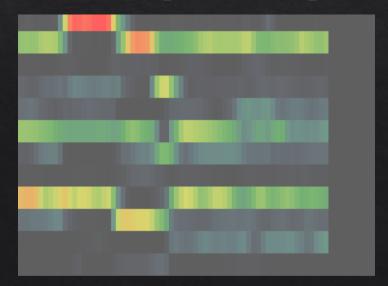
Handling audio inputs

- ◆ Compute *chroma features* of the raw audio
 - ♦ NNLS-Chroma algorithm (Mauch and Dixon, 2010)
- ♦ For each chroma frame
- 1. If the chroma frame vector is very similar to the previous frame, skip it
- 2. Compare the energy in every chroma bin with a *threshold*, if the energy in the chroma bin is higher than the threshold, send it as a pitch class input to the model



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- ♦ We obtain this *global* key by re-analyzing the segmentation of local keys that we obtained from the HMM algorithm
- ♦ The process is very similar to the first HMM algorithm except that now the local keys (as opposed to pitch classes) become the new observations of the model

Results

- ♦ We evaluated our model using the dataset derived by Albrecht and Shanahan (2008)
 - ♦ 982 symbolic music files encoded in Humdrum (**kern) format
 - ♦ Mostly common-practice Western music
 - ♦ All files converted to MIDI and processed as MIDI files
- ♦ Cross-validation through 20 random permutations of the dataset
 - ♦ 490 files for training
 - ♦ 492 files for testing

Results

Table 3: Accuracy Ratings for Key-Finding Methods in Major, Minor, and Overall pieces

Algorithm	Major	Minor	Overall
Krumhansl-Schmuckler	69.0%	83.2%	74.2%
Temperley (with KS algorithm)	96.8%	74.3%	88.6%
Bellman-Budge	94.9%	84.4%	91.1%
Aarden-Essen	90.7%	93.3%	90.4%
Sapp	92.3%	87.2%	90.4%
Albrecht-Shanahan1	92.7%	85.5%	90.0%
Albrecht-Shanahan2	89.1%	95.0%	91.3%
justkeydding (KK)	79.5%	76.3%	78.4%
justkeydding (AE)	86.1%	89.9%	87.5%
justkeydding (S)	89.2%	87.4%	88.5%
justkeydding (BB)	94.3%	81.1%	89.5%
justkeydding (T)	94.2%	83.3%	90.2%
justkeydding (AS)	93.1%	88.1%	91.3%

Combining the key profiles

- ♦ We observed that different key profiles (hyper-parameters) misclassify in different pieces
- ♦ We decided to train a model based on the predictions from different hyper-parameters
- ♦ We used a logistic regression classifier for training this model

Results (with meta-classifier)

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justkeydding (AS)	93.1%	88.1%	91.3%
justkeydding (meta-classifier)	96.1%	91.5%	94.4%

Results

- ♦ The meta-classifier model performed better in overall keys than the state-of-the-art symbolic key detection, using the same dataset
- However, it does a better job predicting the key on a musical piece if it is in a major key than if it is in a minor key, unlike the state-of-the-art algorithm

Results on audio

- ♦ As a further experiment, we synthesized the entire dataset and repeated the experiment on the audio-synthesized version
- In this evaluation, we obtained
 - ♦ 96% accuracy for pieces in a major key
 - ♦ 90.9% accuracy for pieces in a minor key
 - ♦ 94.2% overall accuracy
- Performance is very similar in the audio-synthesized version of the dataset

MIREX 2019

MIREX2005Key

Algorithm	Mirex	Correct	Adjacent	Relative	Parallel	Chromatic	Other
CG1	76.67	67.41	10.54	11.58	2.56	0.32	7.59
CN1	88.07	82.75	8.31	2.72	1.76	0.08	4.39
CN2	89.19	84.19	7.43	3.04	1.92	0.16	3.27
JXC1	86.53	77.56	15.34	3.51	1.28	0.08	2.24
JXC2	88.45	81.07	12.46	3.19	0.96	0.08	2.24

PresegmentedKeyIsophonics

Algorithm	Mirex	Correct	Adjacent	Relative	Parallel	Chromatic	Other
CG1	49.66	37.47	9.66	14.71	14.71	1.38	22.07
CN1	64.99	55.40	13.56	4.60	7.13	3.91	15.40
CN2	74.05	65.52	11.95	4.83	5.52	2.30	9.89
JXC1	79.68	74.02	7.36	5.06	2.30	2.53	8.74
JXC2	79.66	74.94	5.52	4.83	2.53	4.14	8.05

GiantStepsKey

Algorithm	Mirex	Correct	Adjacent	Relative	Parallel	Chromatic	Other
CG1	59.74	49.50	14.24	6.95	5.13	3.81	20.36
CN1	50.53	39.74	11.92	13.24	4.30	2.65	28.15
CN2	57.85	47.52	11.26	11.92	5.63	1.66	22.02
JXC1	65.61	57.45	8.61	9.11	5.63	1.82	17.38
JXC2	75.81	69.04	8.94	5.79	2.81	2.65	10.76

PresegmentedKeyRobbieWilliams

A	lgorithm	Mirex	Correct	Adjacent	Relative	Parallel	Chromatic	Other
Г	CG1	64.97	53.97	13.23	12.17	3.70	0.00	16.93
Г	CN1	62.28	53.97	10.58	7.94	3.17	7.41	16.93
Г	CN2	73.23	64.02	9.52	11.64	4.76	0.00	10.05
Г	JXC1	84.76	78.31	4.76	13.23	0.53	0.00	3.17
	JXC2	82.06	73.55	8.99	11.64	2.65	0.00	3.17

Billboard2012Key

Algorithm	Mirex	Correct	Adjacent	Relative	Parallel	Chromatic	Other
CG1	49.86	36.30	10.96	13.70	19.86	0.00	19.18
CN1	67.40	58.22	12.33	5.48	6.85	6.85	10.27
CN2	73.29	63.70	13.01	6.16	6.16	1.37	9.59
JXC1	85.48	78.08	12.33	2.74	2.05	0.00	4.79
JXC2	90.41	86.99	4.11	1.37	4.79	0.68	2.05

Summary

- ♦ We consider that this algorithm can be useful for segmenting musical pieces in the symbolic and audio domain by their *local keys*
- The model can also be used as global key detection algorithm, providing state-of-the-art performance in the symbolic domain
- ♦ Further work requires the evaluation of local keys and their relationship with musictheoretical concepts such as modulation and tonicization



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Centre for Interdisciplinary Research in Music Media and Technology









compute calcul canada canada



Thank you

https://github.com/napulen/justkeydding

Local keys

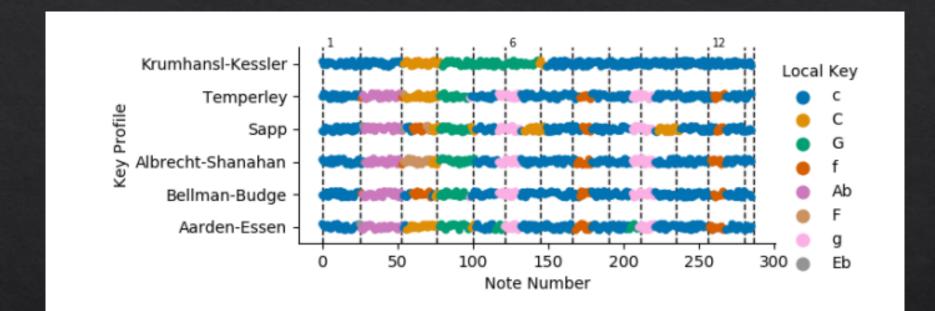


Figure 4: Output of the local key segmentation in Prelude Op.28 No.20 in C minor by Frédéric Chopin. Measures are delimited by dashed vertical lines.

Structural-based analyses

How they start

Perfect and imperfect consonant intervals constitute a consonant interval. Every other is a dissonant interval

A few drinks hours later

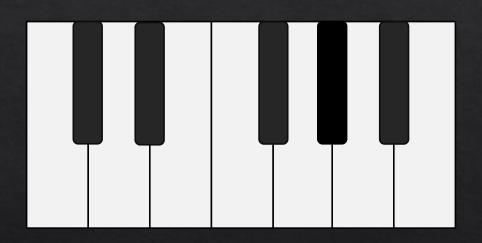
If the goal chord falls on a strong beat and it is a major triad or major-minor seventh, and the root movement from the pre-cadence is an ascending or descending perfect fifth or major second or a descending minor second, and when the root motion is a descending fifth, the pre-cadence is not a potential dominant, and when the root motion is an ascending fifth the pre-cadence is triadic, then the pseudo-cadence is a half cadence, and its strength increases by 10

(Maxwell, 1984)

Jean-Philippe Rameau's modulation

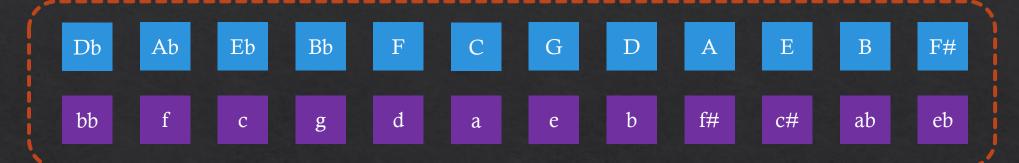
[...] No matter in what key we begin, we should modulate in this key for at least 3 or 4 measures. This number of measures may be exceeded whenever ability and good taste so dictate [...] The ear does not respond with pleasure to a key which is heard too often. The initial key may return from time to time, but as for the others, we should not return immediately to a key we have just left. For example, if I began in the key of Do, I could return to it after having passed through another key. It would be poor, however, later to return to some other key, after having left it to take up the key of Do or another key. It would be preferable to pass into a new key, and then to follow along from one key to another with discretion, returning imperceptibly to those keys which are most closely related to the initial key, finishing there in such a way that it appears as if this key had never been left. After having passed through several other keys, we must modulate in this principal key for a little longer towards the end than at the beginning

Ab Eb Bb F# Db b f# c# d ab bb g a eb C e

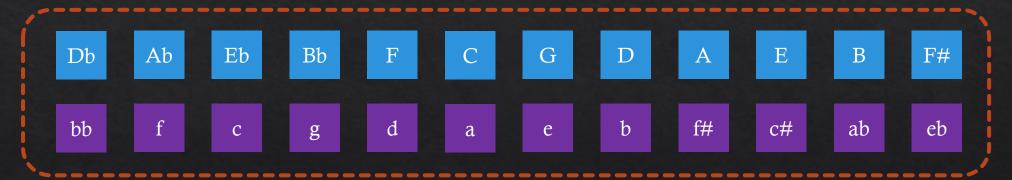








Hidden states (global key)



Observations (local keys)